Analog Computer Applications
Application Note #4
Undergraduate Engineering Physics Electro-Optics STEM Project

Green Light-Emitting Diode (LED) Transient Current Simulation

1 Introduction

This application note addresses the issue of determining the transient current in a DC series circuit that includes an LED, which is a non-linear electro-optical component. Without an LED in the circuit, a differential equation with its explicit analytical solution (by inspection) are shown below:

$$\dot{I} = -I/(RC) \text{ with } I(0) = V_S/R$$

and

$$I = V_S/R\ e^{\wedge}(-t/(RC))$$

However, with an LED in the circuit, a differential equation, for which there is no explicit analytical solution is developed in section 2. Analog computation to the rescue!

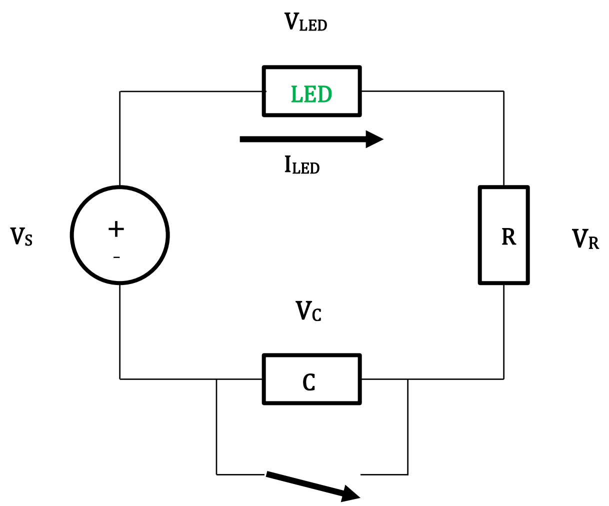

Figure 1: LED-resistor-capacitor series circuit

2 Mathematical modeling

Starting with the ideal light-emitting diode equation,

$$I_{LED} = I_{RS}[e^\wedge(V_{LED}/(\eta kT/q) - 1] \quad (1)$$

where

I_{RS} = diode reverse saturation current
η = ideality factor
k = the Boltzmann constant
T = LED junction temperature
q = electron charge magnitude

Letting $V^* = \eta kT/q$,

$I_{LED} = I_{RS}[e^\wedge(V_{LED}/V^*) - 1]$

Solving for V_{LED},

$$V_{LED} = V^*[\ln(I_{LED} + I_{RS}) - \ln(I_{RS})] \quad (2)$$

Differentiating (2) with respect to t,

$$dV_{LED}/dt = V^*/(I_{LED} + I_{RS})\, dI_{LED}/dt \quad (3)$$

According to DC circuit theory (see figure 1),

$V_R + V_C + V_{LED} = V_S$

$$RI_{LED} + Q/C + V^*[\ln(I_{LED} + I_{RS}) - \ln(I_{RS})] = V_S \quad (4)$$

Differentiating (4) with respect to t,

$RdI_{LED}/dt + 1/C\, dQ/dt + V^*/(I_{LED} + I_{RS})dI_{LED}/dt = 0$ or

$R\dot{I}_{LED} + 1/C\, I_{LED} + V^*/(I_{LED} + I_{RS})\, \dot{I}_{LED} = 0$

$R\dot{I}_{LED} + 1/C\, I_{LED} + V^*/(I_{LED} + I_{RS})\, \dot{I}_{LED} = 0$

$(R + V^*/(I_{LED} + I_{RS}))\ \dot{I}_{LED} = -1/C\ I_{LED}$

$(1 + V^*/(R(I_{LED} + I_{RS}))\ \dot{I}_{LED} = -1/(RC)\ I_{LED}$

Letting $\tau = RC$,

$(1 + V^*/(R(I_{LED} + I_{RS})))\ \dot{I}_{LED} = -I_{LED}/\tau$

$\dot{I}_{LED} = -1/\tau \times I_{LED}/(1 + V^*/(R(I_{LED} + I_{RS})))$

$$\dot{I}_{LED} = -1/\tau \times I_{LED}(I_{LED} + I_{RS})/(I_{LED} + I_{RS} + V^*/R) \text{ with } I_{LED}(0) = I_{LED0} \quad (5)$$

Letting

$\tau = RC = 3.3\ k\Omega \times 470\ \mu F = 1.55\ s$,

$I_{RS} = 0$ mA,

$I_{LED}(0) = 2.50$ mA,

$V^*/R = 0.050$ mA, and

$I_{LED} = I$, then

$$\dot{I} = -I^2/(1.55\ s(I + 0.050\ mA)) \text{ with } I(0) = 2.50\ mA \quad (6)$$

Suppressing units,

$$\dot{I} = -I^2/(1.55(I + 0.050)) \text{ with } I(0) = 2.50 \quad (7)$$

3 Computer setup

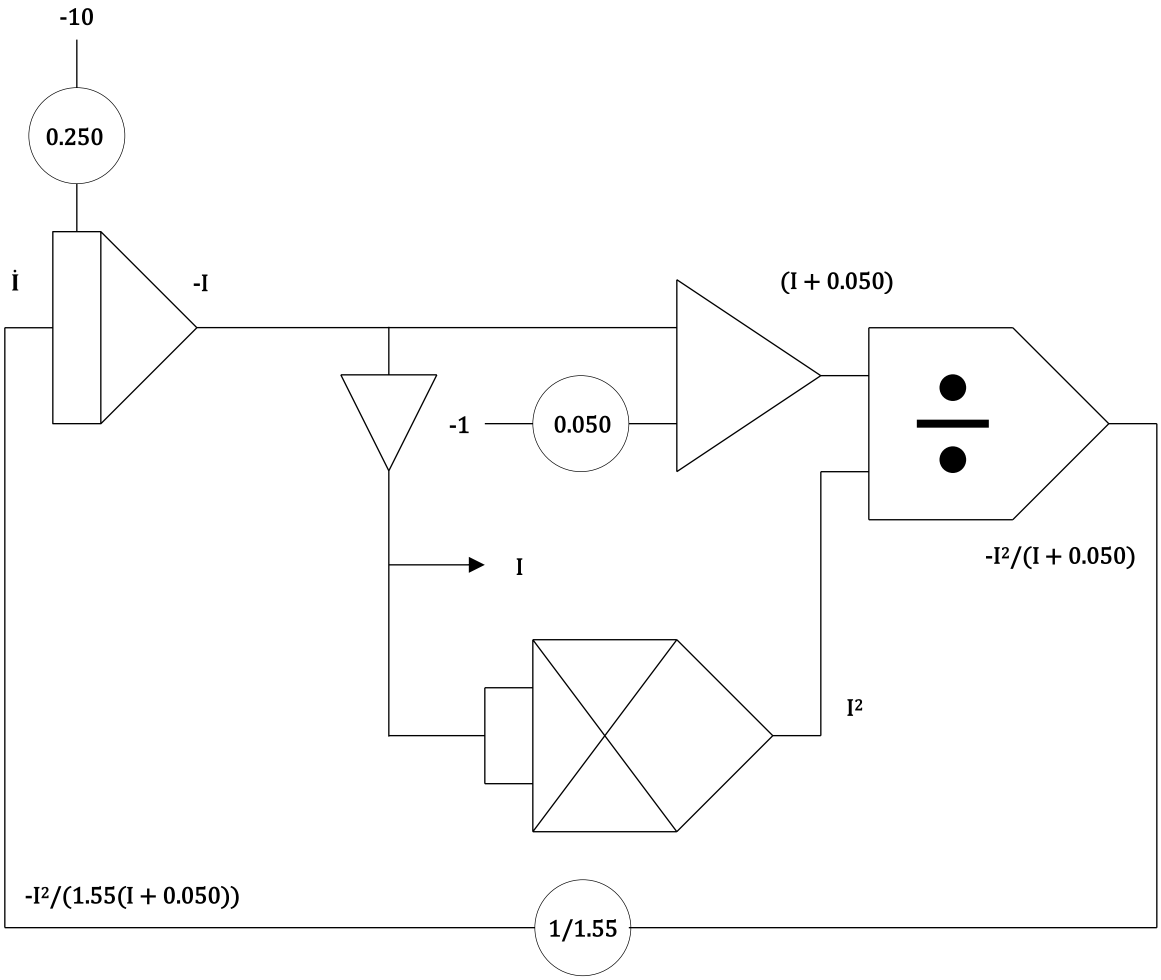

Figure 2: Computer setup for green LED transient current simulation

4 Results

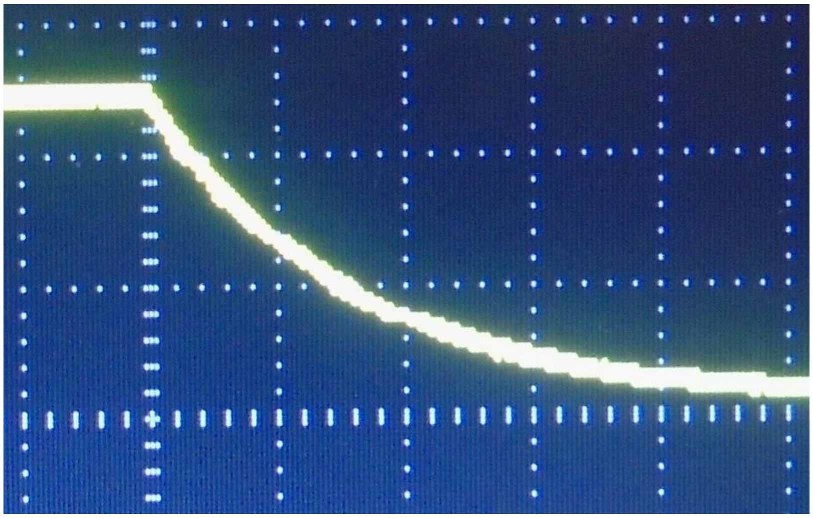

Figure 3: Green LED transient current vs time*

*For this application note, the display was produced during a single run by a differential equation analog computer prototype using discrete components with tolerances between 1% and 10%.

Michael Cimorosi, Issue #4, 18-NOV-2020