Analog Computer Applications
Application Note #5
Undergraduate Engineering Physics STEM Project

Newton's Law of Heating Simulation
(Part 1: Constant Ambient Temperature)

1 Introduction

This simulation concerns itself with the temperature of a glass of iced tea, initially at 1^{o}C, after it is placed in a refrigerator with a constant interior ambient temperature of 6^{o}C. For this simulation, the object's temperature is assumed to obey Newton's Law of Heating.

2 Mathematical modeling

Starting with Newton's Law of Heating,

$$dT/dt = 1/2/\text{min}\ (6^{o}C - T) \text{ with } T(0) = 1^{o}C \tag{1}$$

(Elected to use simple fractions instead of dealing with decimals.)

Replacing dT/dt with $\dot{T}$ and omitting units for clarity,

$$\dot{T} = 1/2\ (6 - T) \text{ with } T(0) = 1 \tag{2}$$

Using differential equation techniques,

$$T = -5e\text{^}(-t/2) + 6 \tag{3}$$

This analytical solution will be compared with the analog computation solution on page 3.

3 Computer setup

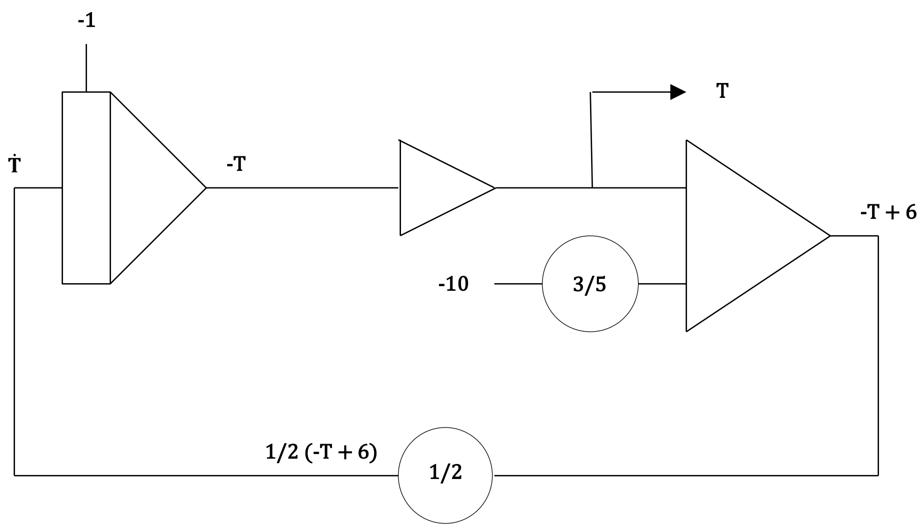

Figure 1: Computer setup for Newton's Law of Heating Simulation (T_A = constant)

4 Results

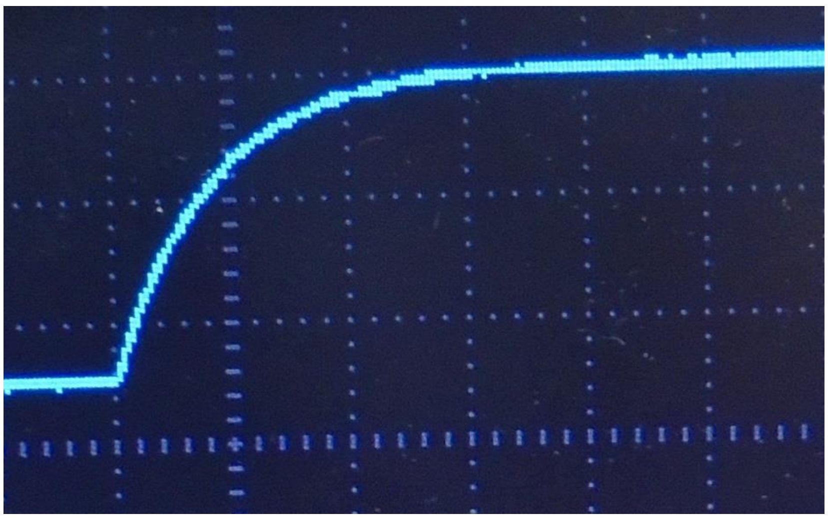

Figure 2: Temperature vs time simulation (constant ambient temperature)*

Analog Computation t(s) Oscilloscope Scale: 0.5 min per small division	Analog Computation T(^{o}C) Oscilloscope Estimate Scale: 0.4^{o}C per small division	Analytical T(^{o}C) Calculator Rounded to two decimal places
0.00	1.0	1.0
5.00	5.6	5.6
10.0	6.0	6.0 = T_{max}

Table 1: Solution Comparison: Analog Computation vs Analytical

Newton's Law of Heating Simulation (Part 2: Linearly Decreasing Ambient Temperature)

1 Introduction

This part of the simulation concerns itself with the temperature of a glass of iced tea, initially at 1^{o}C, after it is placed in a refrigerator with an interior ambient temperature initially at 6^{o}C but then begins to decrease linearly. For this simulation, the object's temperature is assumed to obey Newton's Law of Heating.

2 Mathematical modeling

Starting with Newton's Law of Heating and T_A = (-1/3 ^{o}C/min t + 6^{o}C),

$$dT/dt = 1/2/\text{min}\ (-1/3^oC/\text{min}\ t + 6^oC - T) \text{ with } T(0) = 1^oC \qquad (1)$$

(Elected to use simple fractions instead of dealing with decimals.)

Replacing dT/dt with $\dot{T}$ and omitting units for clarity,

$$\dot{T} = 1/2\ (-t/3 + 6 - T) \text{ with } T(0) = 1 \qquad (2)$$

Rewriting (2),

$$\dot{T} + T/2 = -t/6 + 3 \qquad (3)$$

First, seek a complementary solution (T_c) by setting the right-hand side of (3) equal to zero.

$$\dot{T}_c + T_c/2 = 0$$

$$\dot{T}_c = -T_c/2$$

Integrating by inspection,

$$\ln(T_c) = -t/2 + \text{constant}$$

Setting the constant = $\ln(\alpha)$,

$$\ln(T_c/\alpha) = -t/2$$

$$T_c = \alpha e\text{^}(-t/2) \qquad (4)$$

Next, seek a particular solution (T_p) by selecting a trial function.

Assuming $T_p = \beta t + \gamma$

$$\dot{T}_p = \beta \qquad (5)$$

Substituting the expressions for (4) and (5) into (3),

$$\beta + (\beta t + \gamma)/2 = -t/6 + 3$$

$$\beta/2\ t + \beta + \gamma/2 = -t/6 + 3 \qquad (6)$$

Comparing coefficients of like terms on each side of (6),

$$\beta/2 = -1/6$$

$$\beta = -1/3$$

and

$$\beta + \gamma/2 = 3$$

$$-1/3 + \gamma/2 = 3$$

$$\gamma = 20/3$$

Thus,

$$T_p = -t/3 + 20/3$$

and

$$T = T_c + T_p = \alpha e\text{\^{}}(-t/2) - t/3 + 20/3$$

Invoking the initial condition T(0) = 1,

$$1 = \alpha + 20/3$$

$$\alpha = -17/3$$

yielding

$$T = -17/3\ e\text{\^{}}(-t/2) - t/3 + 20/3 \qquad (6)$$

This analytical solution will be compared with the analog computation solution on page 7.

Differentiating (3) and setting it equal to zero will yield T_{max}.

$$\dot{T} = 17/6\ e\text{\^{}}(-t/2) - 1/3$$

$$0 = 17/6\ e\text{\^{}}(-t'/2) - 1/3$$

Using algebra,

$$t' = -2\ln(2/17) \cong 4.28$$

and

$$T_{max} \cong -17/3\ e\text{\^{}}(-4.28/2) - 4.28/3 + 20/3 \cong 4.57$$

3 Ambient temperature generator computer setup

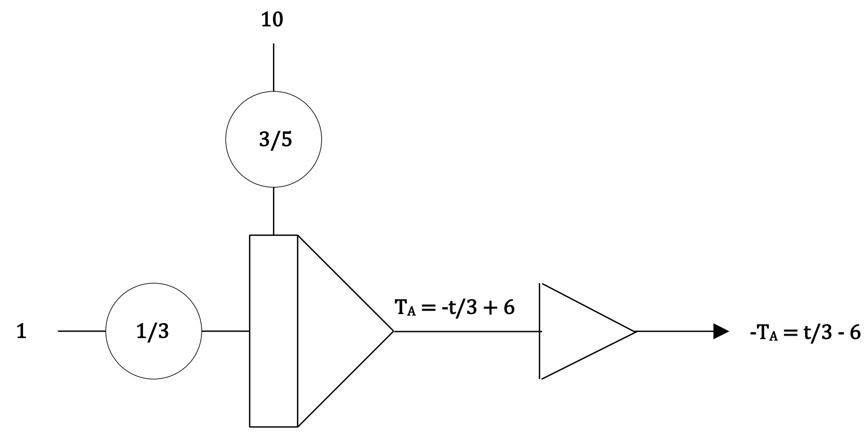

Figure 1: Computer setup to generate decreasing ambient temperature

4 Final computer setup

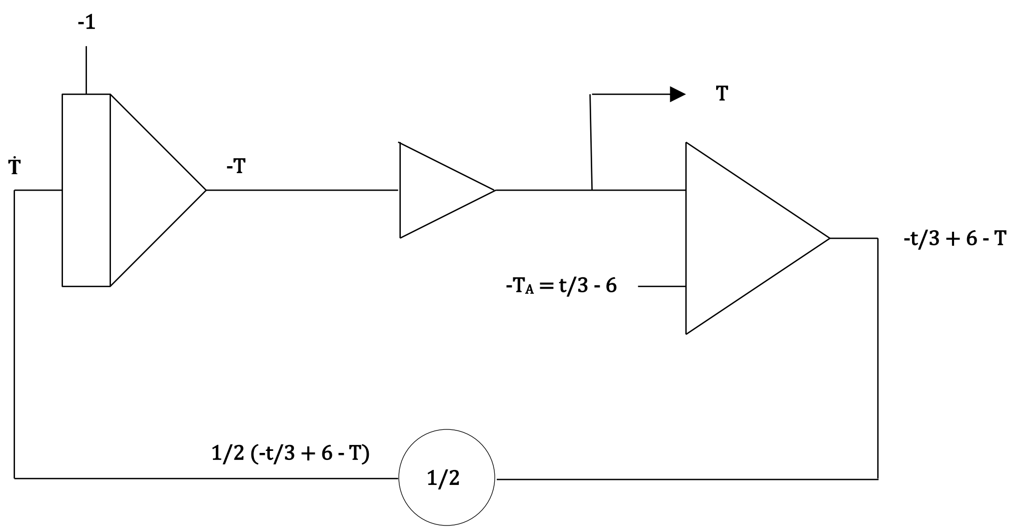

Figure 2: Computer setup for Newton's Law of Heating Simulation

5 Results

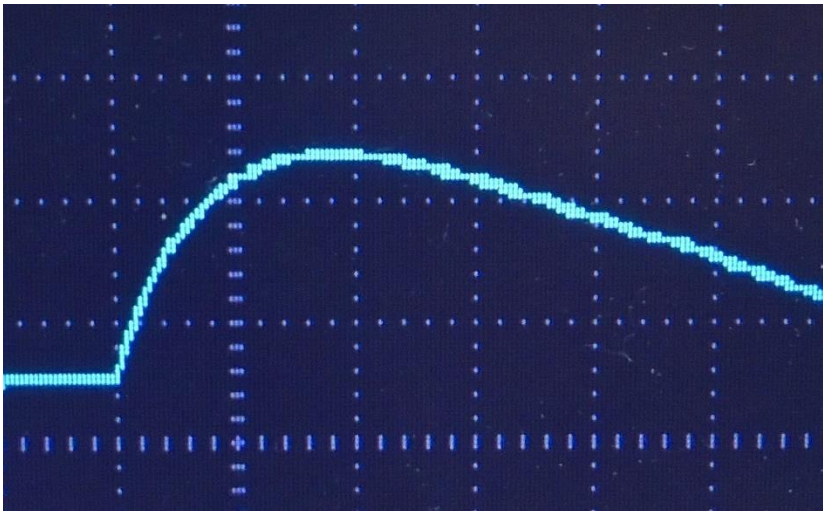

Figure 3: Temperature vs time simulation (decreasing ambient temperature)*

Analog Computation t(s) Oscilloscope Scale: 0.5 min per small division	Analog Computation T(°C) Oscilloscope Estimate Scale: 0.4°C per small division	Analytical T(°C) Calculator
0.00	1.0	1.0
4.28 = t’	4.8	4.6 = T_{max}
10	3.6	3.3

Table 1: Solution Comparison: Analog Computation vs Analytical

*For this application note, each display was produced during a single run by a differential equation analog computer prototype using discrete components with tolerances between 1% and 10%.