Analog Computer Applications
Application Note #6
Undergraduate Engineering Physics Space-Themed STEM Project

Power Surfing on Ligeia Mare, a Large Hydrocarbon Sea on Titan (A Simulation)

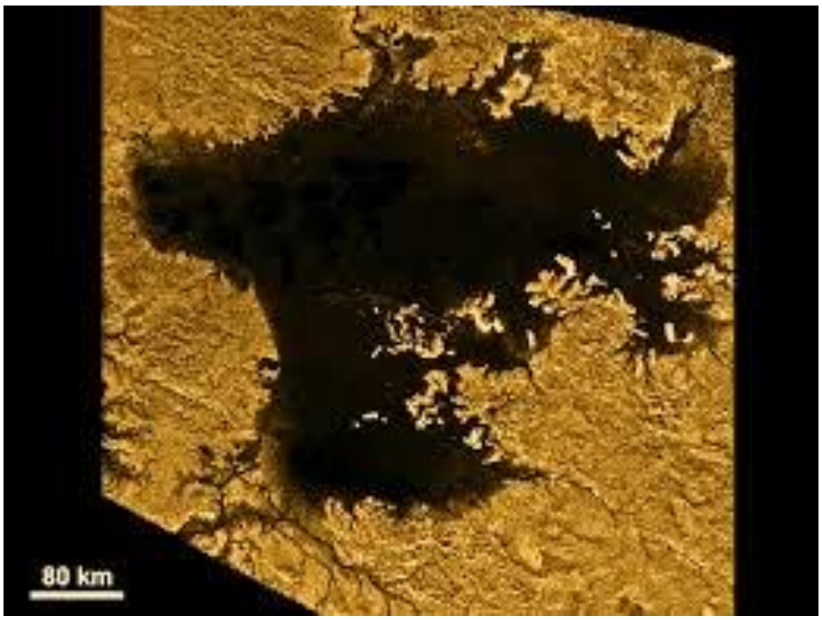

Figure 1: Ligeia Mare (https://en.wikipedia.org/wiki/Ligeia_Mare)

1 Introduction

This application note simulates the speed of a small sea-going drone that is power surfing across Ligeia Mare (from Greek mythology). To start, it is assumed that the drone is moving with a uniform speed of three meters per second. After its electric engine is cut off, the drone begins coasting to a halt under the influence of atmospheric drag (assumed to be quadratic) and surface friction. The drone's speed and the time it takes for it to come to a halt will be determined and displayed in figure 3.

In section 2, a mathematical model will be developed for the drone's speed as a function of time.

For this application, an analytical solution, a numerical method solution, and an analog computation solution will be determined and then compared in table 1.

2 Mathematical modeling

Starting with Newton's second law of motion,

$$ma = F_{net} = -drag - friction = -kv^2 - \mu mg$$

Assuming m = 1 kg, k = 0.1 kg/m, g = 1.352 m/s^2, μ = 0.125 (just a guess), v(0) = 3 m/s, and neglecting units for simplicity,

$$a = -0.1v^2 - 0.169 \text{ with } v(0) = 3$$

Since a = dv/dt,

$$dv/dt = -0.1v^2 - 0.169 \text{ with } v(0) = 3 \quad (1)$$

Equation (1) is separable and integrable. Additional mathematical details provided upon request: mcimorosi@desu.edu

$$dv/dt = -0.1(v^2 + 1.69)$$

$$dv/dt = -0.1(v^2 + 1.3^2)$$

$$\int_3^v \frac{dv}{v^2 + 1.3^2} = -0.1\int_0^t dt$$

$$[\tan^{-1}(v/1.3) - \tan^{-1}(3/1.3)]/1.3 = -0.1t \quad (2)$$

$$v = 1.3\tan[\tan^{-1}(3/1.3) - 0.13t] \quad (3)$$

Setting v = 0 m/s in (2) to determine the time required to come to a halt,

$$t_{halt} = \tan^{-1}(3/1.3)/0.13 = 8.94 \text{ seconds} \quad (4)$$

3 Computer setup

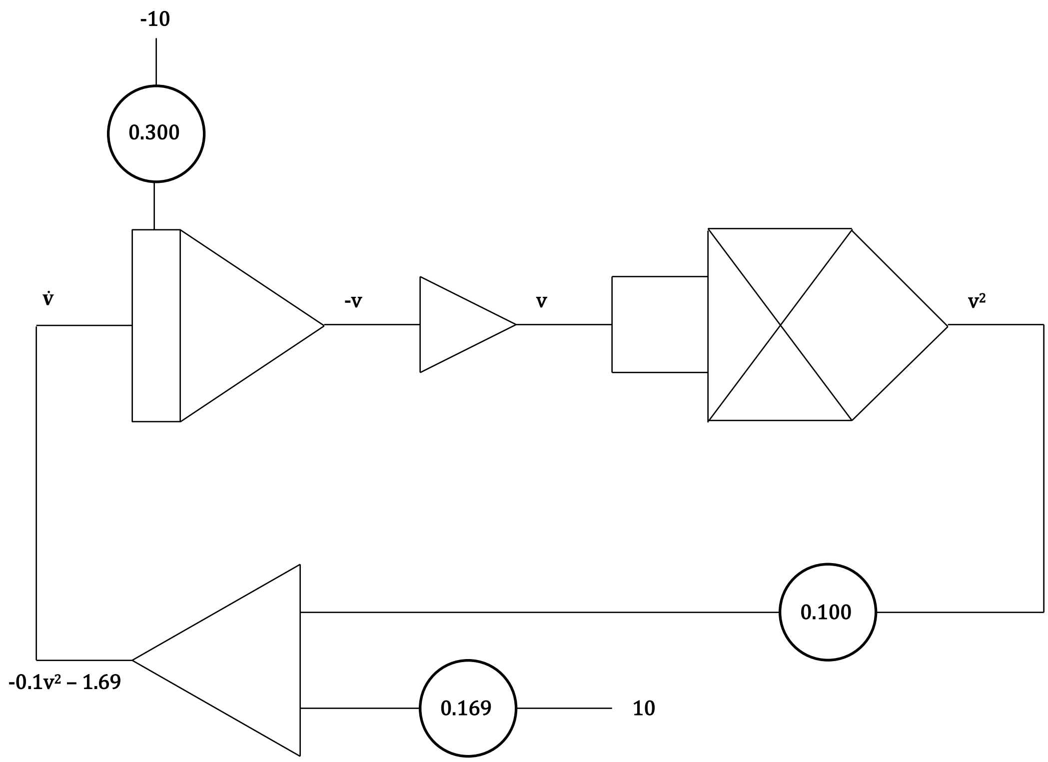

Figure 2: Computer setup for coasting across the surface of a Ligeia Mare

Heun's Numerical Method
(hand-held programmable calculator)

```
PROGRAM:TITANV
:ClrHome:ClrDraw:Fix 2
:"DV/DT=-0.1(V² + 1.69)"
:"INITIAL CONDITION"
:0→T:3→V
:"STEP SIZE":0.05→H
:Lbl 1
:If V<-0.05:Then
:Goto 2:Else
:Disp {T,V}
:-0.1(V² +1.3²)→F
:T+H→T
:V+FH→W
:-0.1(W² +1.3²)→S
```

```
:(F+S)/2→A
:V+AH→V
:Pause:Goto1
:Lbl2:End
```

4 Results

t(s)	Analog Computation Solution v(m/s)	Analytical Solution v(m/s)	Heun's Numerical Method Solution H = 0.05 s v(m/s)
0.00	3.00	3.00	3.00
2.50	1.50	1.44	1.44
5.00	0.80	0.73	0.73
7.50	0.30	0.25	0.25
9.50	0.00		
8.94		0.00	
8.95			0.00

Table 1: Solution Comparisons

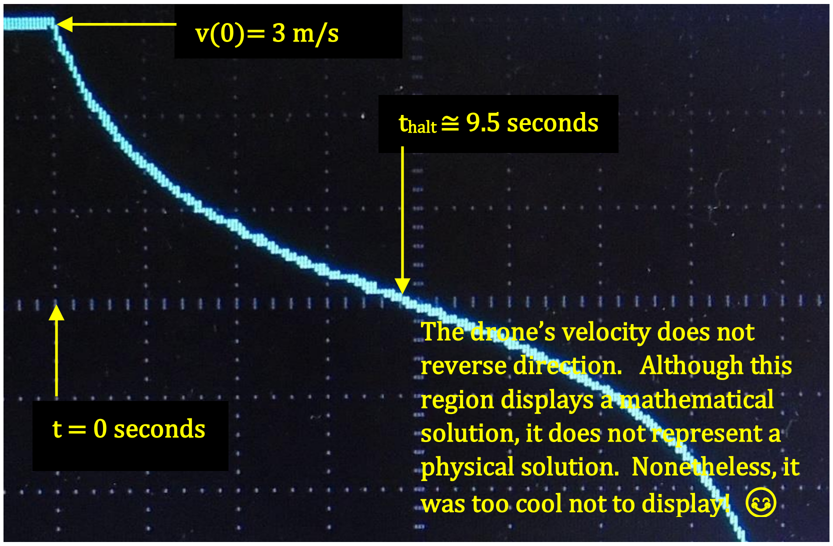

Figure 3: Drone speed as a function of time*

*For this application note, the display was produced during a single run by a differential equation analog computer prototype using discrete components with tolerances between 1% and 10%.