Sprott SQ_M model

This application note implements the SQ_M model (cf. [Sprott(2016), pp. 68ff.]), an example of a simple three-dimensional chaotic flow with a quadratic nonlinearity. The model consists of three coupled differential equations

$$\dot{x} = -z$$
$$\dot{y} = -x^2 - y$$
$$\dot{z} = \alpha + \alpha x + y$$

with $\alpha = 1.7$ and the initial conditions $x(0) = 1, y(0) = -0.8, z(0) = 0$. To scale the system, three scale factors $\lambda_x = \lambda_z = \frac{1}{4}$ and $\lambda_y = \frac{1}{6}$ are introduced which in turn yield (after collecting all resulting factors)

$$\dot{x} = -z \tag{1}$$
$$\dot{y} = -2.666x^2 - y \tag{2}$$
$$\dot{z} = \frac{\alpha}{4} + \alpha x + 0.15y. \tag{3}$$

Thanks to the constant term $\frac{\alpha}{4}$, the initial conditions mentioned in the original system can be safely ignored as it will enter its chaotic oscillation right away.

The analog computer setup can be derived directly from the scaled equations (1), (2) and (3) as shown in figure 1.

This little program is ideally suited for *THE ANALOG THING* as can be seen in figure 2. Figure 3 shows a typical phase space plot of x vs. y which was caputured using a USB-soundcard with stereo line in and the software *Oscilloppoi*[1] running on a Mac.

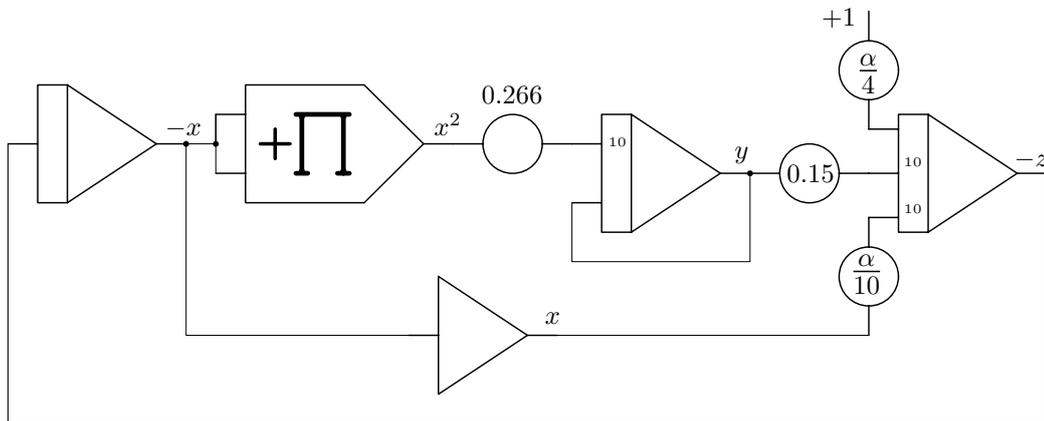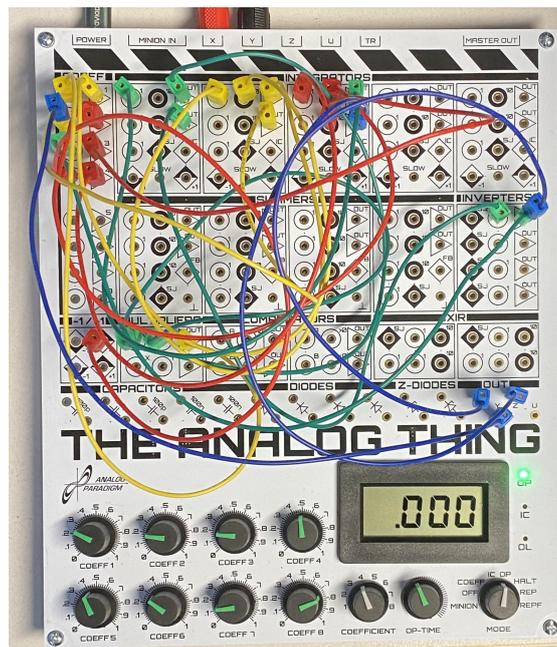[1]See https://anikikobo.com/software/oscilloppoi/index_en.html.

Figure 1: Analog computer setup for the SQ_M model

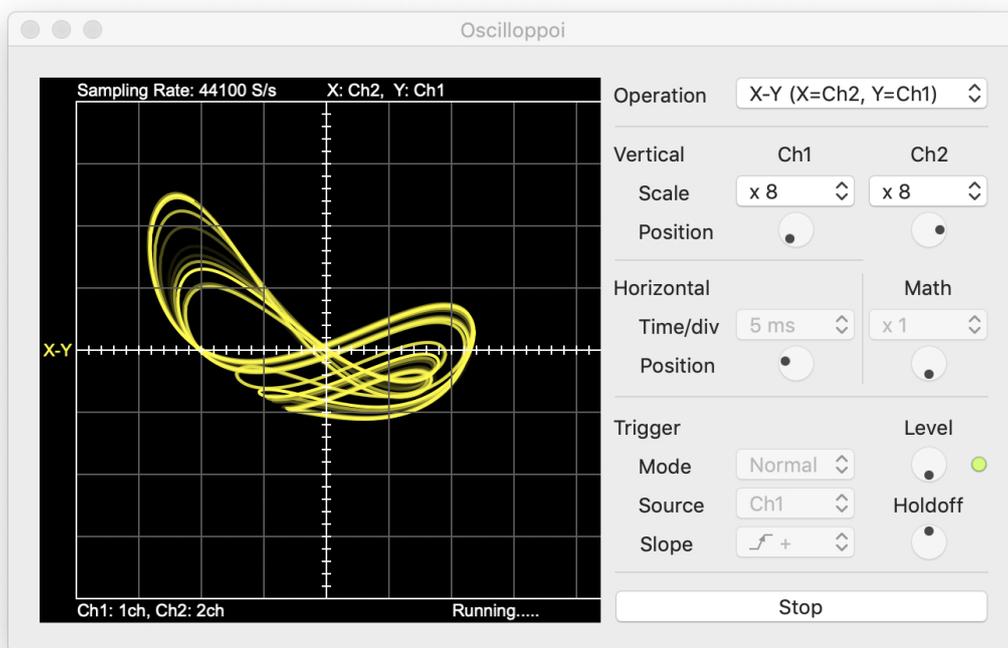Figure 2: Implementation of the program shown in figure 1

Figure 3: xy phase space plot of the SQ_M system

References

[SPROTT(2016)] JULIEN CLINTON SPROTT, *Elegant Chaos – Algebraically Simple Chaotic Flows*, World Scientific, 2016