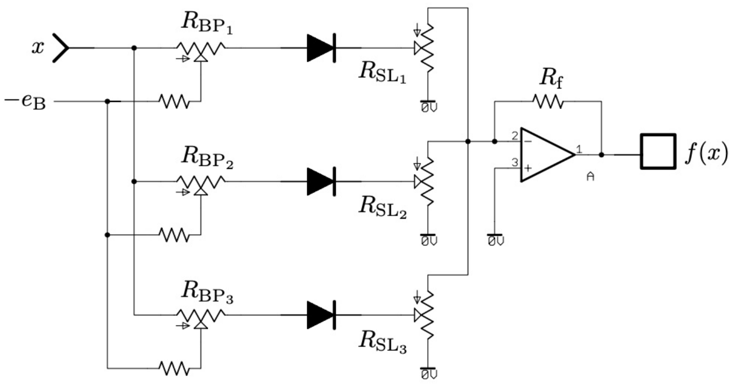

Figure 1: Principle of operation of a diode function generator (see [ULMANN 2023, p. 101])

A segmented-sigmoid-function (SSF)

1 Introduction

Sometimes one would like to have approximations for functions such as $\sin(x)$, $\cos(x)$, or $\tanh(x)$ available on an analog computer. This requires x to be bounded by $-1 \leq x \leq 1$. In the following an easy way to implement a *segmented-sigmoid-function* (*SSF*) is shown.

The easiest approach to such approximations is a diode function generator as shown in figure 1. The idea is to approximate a function by a polygonal line with appropriately biased diodes controlling the slopes of the segments of the polygonal line. Classic diode function generators typically have either eleven segments with adjustable break points (EAI and others) or 21 segments with equidistant breakpoints (Telefunken). The idea is quite straightforward: The potentiometers R_{BP_i} set the position of the breakpoints while the potentiometers R_{SL_i} control the slope of the corresponding segment. Setting such function generators up is a rather tedious process when done manually and the resulting precision is typically of the order of 1%.

Dr. BERND ULMANN
Issue #58, 26-NOV-2025

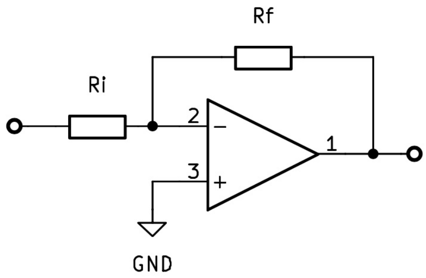

Figure 2: Basic inverting amplifier setup

2 A much simpler approach

Figure 2 shows a basic inverting amplifier consisting of an operational amplifier in inverting configuration with an input resistor R_i and a feedback resistor R_f. The quotient R_f/R_i determines the gain which is 1 for $R_\text{f} = R_\text{i}$. Adding a diode (ideally a SCHOTTKY-diode due to its small forward voltage) in parallel to R_f as shown in figure 3 basically implements a *RELU* (short for *Rectified Linear Unit*) function. In this example, `D1` conducts when the output of the operational amplifier satisfies $U_\text{out} < 0$, thus limiting the output to nearly 0[1] while it does not conduct for positive output values.

This basic circuit can now be extended to a very simple ZENER-*diode* (or *avalanche diode*) based function generator. ZENER- and avalanche diodes behave like a normal diode when biased forwards but exhibit a pronounced breakdown voltage for reverse biasing. Figure 4 shows the schematic of a very simple five segment ZENER-diode function generator. It requires two pairs of diodes, namely `Z1`, `Z1'`, `Z2`, and `Z2'` with breakdown voltages of U_1 for the first pair and U_2 for the second with $U_1 < U_2$.

In the case $U_\text{out} < U_1$ none of the ZENER-diodes conducts. The gain of the circuit is thus determined by $R_{\text{f}_1}/R_\text{i}$ only. This implements the first segment of the function approximation which is symmetric about 0 with a slope determined by R_f/R_i.

[1] In fact the output is clamped at the forward voltage of the diode.

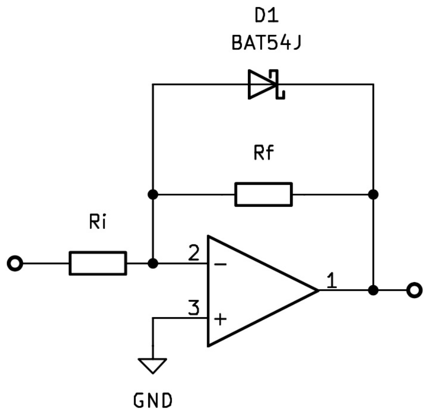

Figure 3: Simple RELU function

When $U_1 \leq U_{\text{out}} < U_2$ the gain is determined by the parallel circuit consisting of R_{f_1} and R_{f_2} since `Z1` and `Z1'` both conduct.[2] This defines the next pair of segments to the left and right of the first segment.

Finally, when $U_{\text{out}} \geq U_2$ the upper pair of diodes will conduct, thus limiting the output voltage of the operational amplifier creating two horizontal line segments on the far left and right of the function.

On an analog a setup like this requires an *open amplifier* which has no built-in feedback resistor R_f. In systems such as anabrid's *Model-1* or *THE ANALOG THING* a summer can be configured to act as an open amplifier by connecting its `FB` jack with ground as shown in figure 5. In general it is advisable to have a rather low-impedance circuit, especially when the bandwidth of the input signal exceeds a few 10 Hz. Accordingly, inputs with weight 10 should be used with the values for `Rf1` and `Rf2` chosen accordingly.

Table 1 shows suitable values for `Rf1`, `Rf2`, and the ZENER-diodes implementing a $\tanh(x)$-approximation using a summer in a Model-1 analog computer or THE ANALOG THING. The behavior of the function is shown in figure 6.[3]

[2] A more thorough analysis must take the voltage drops across the diodes into account which is omitted here.
[3] The diagonal line on the screen is the inverted input value x.

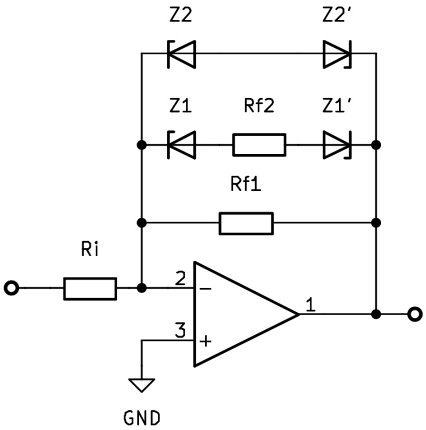

Figure 4: Five segment ZENER-diode function generator

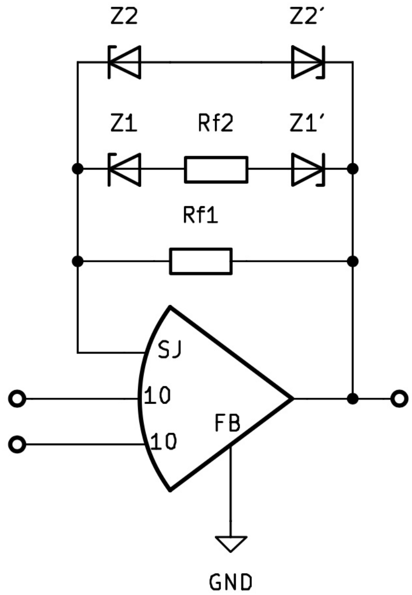

Figure 5: Five segment ZENER-diode function generator using an open amplifier on an analog computer

	Rf1	Rf2	Z1,Z1'	Z2,Z2'
Model-1	100k	30k	6.7 V	10 V
THE ANALOG THING	470k	100k+47k	6.7 V	10 V

Table 1: Resistor and ZENER-diode values for a $\tanh(x)$-approximation

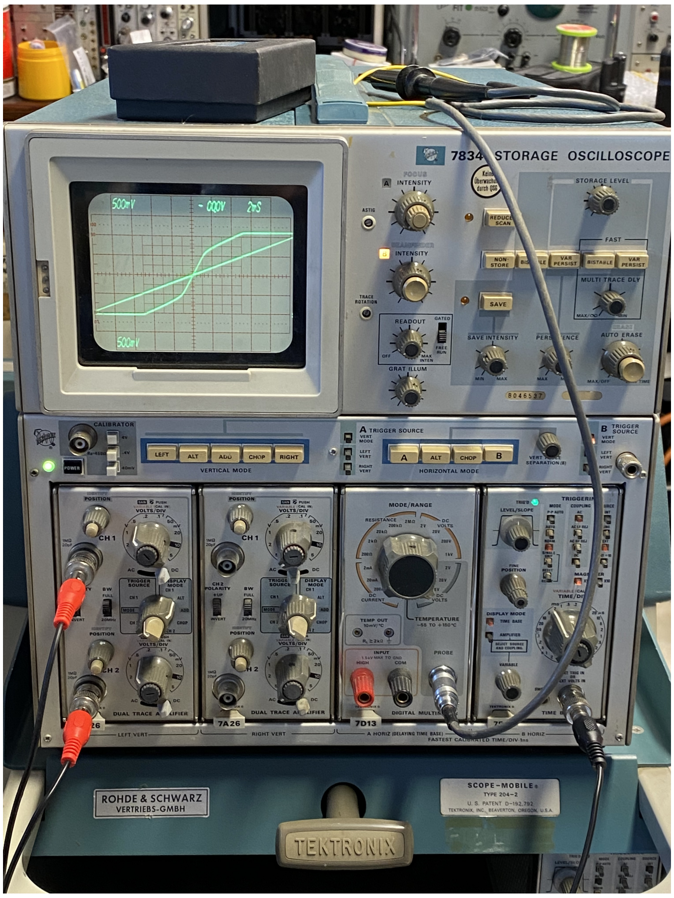

Figure 6: Behavior of the $\tanh(x)$-approximation on THE ANALOG THING

References

[Ulmann 2023] Bernd Ulmann, *Analog Computing*, 2nd edition, DeGruyter, 2023