Mode-Locked Gaussian Laser Power Pulse Simulation

1 Introduction

This application simulates a mode-locked Gaussian laser power pulse without the use of an antilogarithmic computing element.

The first task is to produce a differential equation from a typical Gaussian function. This ‘reverse engineering’ approach is discussed in detail below.

2 Mathematical modeling

Assume a time-shifted mode-locked Gaussian laser power pulse:

\[ P = P_{\text{max}} e^{-4\ln(2)\left(\frac{(t - \tau_s)}{\tau_p}\right)^2} \]  

where

- \( P \) = laser power
- \( P_{\text{max}} \) = maximum (or peak) laser power
- \( t \) = time
- \( \tau_s \) = time shift
- \( \tau_p \) = FWHM (Full Width at Half Maximum)

Letting \( \gamma = \frac{4\ln(2)}{\tau_p^2} \),

\[ P = P_{\text{max}} e^{-\gamma(t - \tau_s)^2} \]

Differentiating \( P \) with respect to \( t \),

\[ \dot{P} = -2\gamma(t - \tau_s)' P_{\text{max}} e^{-\gamma(t - \tau_s)^2} \]

\[ \dot{P} = -2\gamma(t - \tau_s) \]
\[ \dot{P} + 2\gamma(t - \tau_s)P = 0 \text{ with } P(0) = P_0 \]  
\hspace{1cm} (2)

where

\[ \dot{P} = \frac{dP}{dt} \]
\[ \tau_s = \text{time shift} \]
\[ P_0 = \text{laser power at initiation of program run} \]

Equation (2) is the desired differential equation for which the analog computer is well suited to solve using a few operational amplifiers and an analog multiplier.

Letting \( e = 2\gamma(t - \tau_s) \),

\[ \dot{P} + eP = 0 \text{ with } P(0) = P_0 \]  
\hspace{1cm} (3)

Generating \( e \): The operational amplifier (op-amp) integrator to the rescue!

For an op-amp integrator, with a constant input voltage,

\[ V_{out} = -(V_{in}/\tau) t + V_0 = -(V_{in}/\tau)(t - (V_{in}/V_0)\tau) \]

where

\[ V_{in} = \text{input voltage} \]
\[ V_0 = \text{initial voltage} \]
\[ \tau = RC \text{ (time constant)} \]

Letting \( V_{out} = e, V_{in} = \alpha, \text{ and } V_0 = \beta, \)

\[ e = -(\alpha/\tau)(t - (\beta/\alpha)\tau) \]  
\hspace{1cm} (5)

Comparing (5) to (3),

\[ \alpha = 2\gamma \tau = 8\ln(2)\tau / \tau_p^2 \text{ and } \beta = \alpha \tau_s / \tau = 2\gamma \tau \tau_s / \tau = 2\gamma \tau_s = 8\ln(2)\tau_s / \tau_p^2 \]

Letting \( P_0 = 0.100, \tau = 1.760, \tau_p = 8.331, \text{ and } \tau_s = 7.500, \)

\[ \alpha = 0.141 \text{ and } \beta = 0.599 \]
3 Computer setup

Figure 1: Computer setup for mode-locked Gaussian laser power pulse simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.141</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.599</td>
</tr>
<tr>
<td>$P_0$</td>
<td>0.100</td>
</tr>
</tbody>
</table>

Table 1: Parameter settings for mode-locked Gaussian laser power pulse simulation
4 Result

Figure 2: Mode-locked Gaussian laser power pulse*

*For this application note, the display was produced during a single run by a differential equation analog computer prototype using discrete components with tolerances between 1% and 10%.

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