Analog Computer Applications
Application Note #3
Undergraduate Engineering Physics Laser-Themed STEM Project

Destination Europa:
Laser Power-Beaming Simulation

1 Introduction

This application note is based on a hypothetical space mission to explore just beneath the surface of Europa's ocean at a fixed depth. A submersible probe is designed to experience neutral buoyancy (weight = buoyancy). Thus, assuming no external horizontal lateral forces acting on the probe, then only thrust and drag need to be considered. For this simulation, thrust is assumed to be a linear function of laser power. Since a laser is the only available source of power for the probe's port and starboard electric motors (see figure 1), the issue of laser beam transmission through seawater must be considered. The laser power delivered to the photovoltaic array located at the aft end of the probe is assumed to be an exponential function.

Initially, the probe is given a mechanical 'nudge' (not shown in the figure) with a constant speed over a short distance just prior to activating the laser.

This simulation solves and displays the speed of the probe as it travels through this alien ocean!

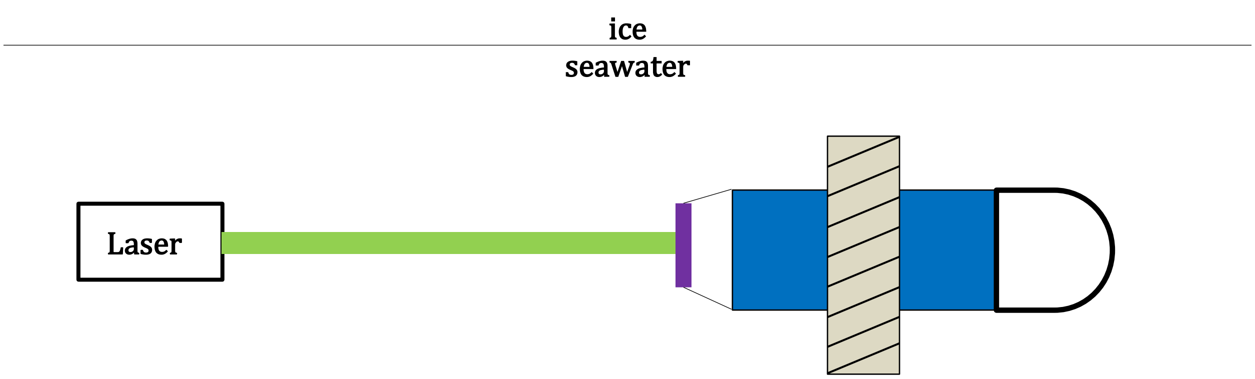

Figure 1: Laser Power-Beaming (rendition)

2 Mathematical modeling

Starting with Newton's second law of motion,

$Ma_x = \sum forces_{x\text{-direction}} = -drag + thrust = -D + T$ (1)

$Ma_y = \sum forces_{y\text{-direction}} = 0$ (no lateral forces present) (2)

$Ma_z = \sum forces_{z\text{-direction}} = -weight + buoyancy = 0$ (neutral buoyancy) (3)

Returning to (1),

$Ma_x = -D + T$

Assuming

$D = ½C_D\rho Av^2 = \kappa v^2$ (quadratic drag model),

$T = cP = cP_{max}e\text{^}(-\alpha x) = T_{max}\, e\text{^}(-\alpha x) = \tau e\text{^}(-\alpha x)$, and

$a_x = dv/dt = dx/dt\ dv/dx = vv'$ (per chain rule for differentiation), then

$Mvv' = -\kappa v^2 + \tau e\text{^}(-\alpha x)$ or

$$v' = (-\kappa/Mv + \tau/Me\text{^}(-\alpha x)/v) \text{ with } v_0 = v(x_0) \quad (4)$$

where

M = mass of probe
x = distance between laser aperture and probe's photovoltaic array
v = speed of probe
κ = drag factor
τ = maximum thrust
α = absorption coefficient

After several trial function 'guesses', the analytical solution takes the form

$$v = \gamma\sqrt{}(x)e\text{^}(-\alpha x/2) \quad (5)$$

Differentiating (5) to obtain the left-hand side of (4),

$v' = \gamma\sqrt{(x)}(-\alpha/2\ )e^{\wedge}(-\alpha x/2)) + e^{\wedge}(-\alpha x/2)(\gamma/(2\sqrt{(x)})$

Equating this expression to the right-hand side of (4),

$\gamma\sqrt{(x)}(-\alpha/2)e^{\wedge}(-\alpha x/2)) + e^{\wedge}(-\alpha x/2)(\gamma/(2\sqrt{(x)}) =$

$(-\kappa/M)\gamma\sqrt{(x)}e^{\wedge}(-\alpha x/2) + (\tau/M)e^{\wedge}(-\alpha x/2)/(\gamma\sqrt{(x)})$

After a bit of algebra,

$(-\alpha/2)x + (1/2) = (-\kappa/M)x + \tau/(M\gamma^2)$

Comparing coefficients of like terms,

$-\alpha/2 = -\kappa/M$ or $M\alpha = 2\kappa$ and

$1/2 = \tau/(M\gamma^2)$ or $\gamma = \sqrt{(2\tau/M)}$

Thus, in general,

$$v = \gamma\sqrt{(x)}e^{\wedge}(-\alpha x/2) \text{ with } v(x_0) = v_0 \text{ and } \gamma = \sqrt{(2\tau/M)} \qquad (6)$$

It is useful, and perhaps instructive, to analytically determine the distance at which the probe acquires maximum speed and what that speed is.

Differentiating (6) with respect to x,

$v' = \gamma\sqrt{(x)}(-\alpha/2)e^{\wedge}(-\alpha x/2) + \gamma e^{\wedge}(-\alpha x/2)/(2\sqrt{(x)})$

Setting $v' = 0$ to determine x^*,

$0 = \gamma\sqrt{(x^*)}(-\alpha/2)e^{\wedge}(-\alpha x^*/2) + \gamma e^{\wedge}(-\alpha x^*/2)/(2\sqrt{(x^*)})$

$0 = -\alpha\sqrt{(x^*)} + 1/\sqrt{(x^*)}$

$x^* = 1/\alpha$ and

$v_{max} = \sqrt{(2\tau/M)}\sqrt{(x^*)}e\text{^}(-\alpha x^*)$

$= \sqrt{(2\tau x^*/M)}e\text{^}(-\alpha x^*/2)$

$= \sqrt{(2\tau/(M\alpha))}e\text{^}(-1/2)$

$= \sqrt{(2\tau/(2\kappa))}e\text{^}(-1/2) = \sqrt{(\tau/\kappa)}e\text{^}(-1/2)$

Letting $\alpha = 0.0641\ m^{-1}$, $\tau = 1.00$ N, $\kappa = 0.0400$ kg/m, and M = 1.247 kg,

$x^* = 1/0.0641 m^{-1} = 15.6$ m, and

$v_{max} = \sqrt{(1.00\ N/0.0400\ kg/m)}e\text{^}(-1/2) = 3.03$ m/s

For the computer setup (figure 2), rewrite (4) as

$v' = (-\kappa v + \tau e\text{^}(-\alpha x)/v)(1/M)$ with $v_0 = v(x_0)$

Parameter calculations

Substituting values into (6),

$v_0 = v(0.5\ m) = \sqrt{(2 \times 1.00\ N/1.247\ kg \times 0.5\ m)}e\text{^}(-0.0641\ m^{-1} \times 0.5\ m/2)$

$= 0.881$ m/s and

$T_0 = T(0.5\ m) = 1.00\ N\ e\text{^}(-0.0641\ m^{-1} \times 0.5\ m) = 0.968$ N

3 Computer setup

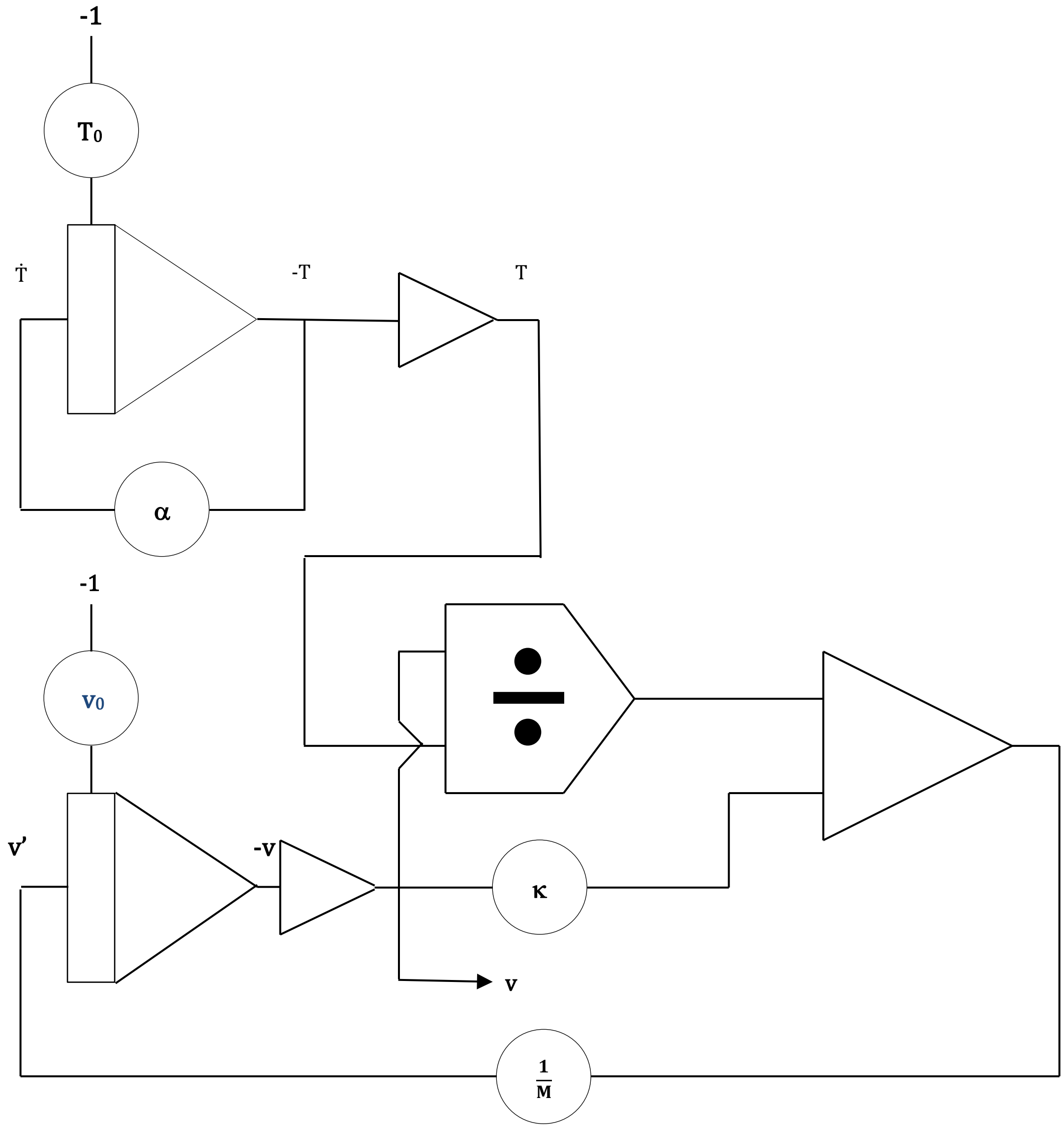

Figure 2: Computer setup for laser power-beaming simulation

Parameter	Value
κ	0.0400
T_0	0.968
α	0.0641
v_0	0.881
1/M	0.802

Table 1: Parameter settings for laser power-beaming simulation

4 Results

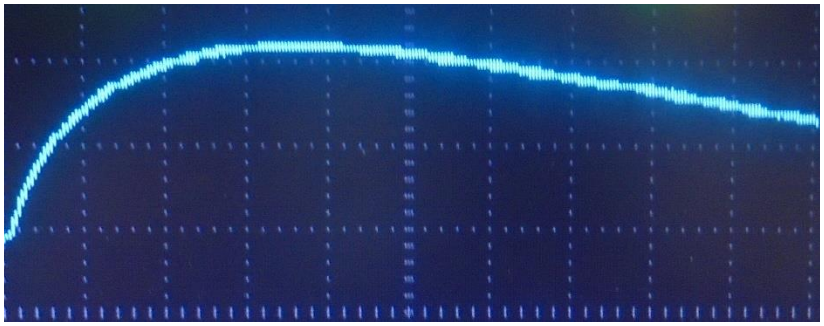

Figure 3: Probe speed as a function of distance traveled*

Analog Computation x (m) Oscilloscope Scale: 1 m per small division	Analog Computation v (m/s) Oscilloscope Scale: 0.2 m/s per small division	Analytical v (m/s) Calculator Rounded to two decimal places
00.5	0.90	0.88 = v_0
15.6	3.15	3.03 = v_{max}

Table 2: Solution Comparison: Analog Computation vs Analytical

*For this application note, the display was produced during a single run by a differential equation analog computer prototype using discrete components with tolerances between 1% and 10%.

Michael Cimorosi, Issue #3, 13-OCT-2020